Anticipating chaotic synchronization

Henning U. Voss*
Institut für Physik, Universität Potsdam, 14469 Potsdam, Germany
(Received 23 December 1999)

Dissipative chaotic systems with a time-delayed feedback can drive near-identical systems in such a way that the driven systems anticipate the drivers by synchronizing with their (arbitrarily distant) future states. This counterintuitive behavior is globally stable, robust, and a pure result of the interplay between delayed feedback and dissipation. Thus it constitutes a rather universal phenomenon of nonlinear dynamics. For small anticipation times, anticipating synchronization also occurs in chaotic systems without a memory term in the driver.

PACS number(s): 05.45.Xt, 05.45.Vx

I. INTRODUCTION

It is well known that dissipative systems with a nonlinear time-delayed feedback or ''memory'' can produce chaotic dynamics [1,2], and the dimension of their chaotic attractors can be made arbitrarily large by increasing their delay time τ sufficiently [3,4]. Recently, it was shown [5] that a time-delayed feedback system

$$\dot{x}=-\alpha x+f(x_\tau) \quad [x\in\mathbb{R},\ x_\tau:=x(t-\tau),\ f \text{ continuous}] \tag{1}$$

can drive an identical system

$$\dot{y}=-\alpha y+f(y_\tau)+K(x-y) \quad (K\in\mathbb{R}), \tag{2}$$

such that both systems are completely synchronized. This means that $x(t)$ equals $y(t)$ for all times t larger than some transient time; ''$x=y$'' is usually called the synchronization manifold [6] of the coupled system. For $x=y$ the coupling term vanishes, and the ''drive system'' (1) and ''response system'' (2) become identical.

In this paper we will consider two other unidirectional coupling configurations; in particular, we will show that (1) the synchronization manifold can turn out to be $x=y_\tau$ [or, equivalently, $y(t)=x(t+\tau)$, meaning that the response system anticipates the driver], and that (2) complete synchronization is also possible if the coupling sets in delayed. We give analytic evidence for both cases (Secs. II and III).

These results hold for arbitrarily large anticipation times. They are generalized to the case of synchronization between coupled chaotic systems described by ordinary differential equations [7,8], i.e., without a time delayed feedback in the driver. In this case the coupling still can be chosen so as to yield an anticipating synchronization manifold, and we give numerical evidence that this manifold can indeed be stable for small anticipation times (Sec. IV). Finally, we briefly discuss some of the most counterintuitive consequences and possible applications of anticipating synchronization (Sec. V).

II. ANTICIPATING SYNCHRONIZATION

The first coupling configuration considered resembles the method of ''complete replacement'' [6], where the driver variable x substitutes a corresponding variable in the response system. In order to achieve the desired synchronization manifold $x=y_\tau$, instead of replacing y by x we replace y_τ by x. This leads to the response system

$$\dot{y}=-\alpha y+f(x) \tag{3}$$

for system (1). For the time evolution of the difference variable $\Delta(t):=x(t)-y(t-\tau)$, one immediately finds $\dot{\Delta}=-\alpha\Delta$. A sufficient synchronization condition in this case is simply

$$\alpha>0. \tag{4}$$

Therefore, the synchronization manifold $x=y_\tau$ is globally attracting and asymptotically stable [9], and after some transient time the system will relax to it. In other words, at time t the response system (3) synchronizes with the *future* state of the driver (1) at time $t+\tau$ and anticipates its dynamics. This result is not only independent of the function f but also of the time delay, such that in principle arbitrarily high-dimensional chaotic motions can literally be predicted over an arbitrary space of time. Since the relaxation time of the difference system depends only on α, transient dynamics play only a minor role, and the onset of synchronization is approached independently of τ. The response system for its part can drive another system, for which it again holds that the difference system evolves like $\dot{\Delta}=-\alpha\Delta$ (assumed that the former systems are already completely synchronized), and so on. Therefore, with a chain of oscillators, the future state of the driver can be predicted for any integer multiple of τ. For this setup, the relaxation rate to the synchronization manifold will decrease not more than linearly with an increasing number of oscillators.

We demonstrate this result on the numerical simulation of two coupled Ikeda equations,

$$\dot{x}=-\alpha x-\beta\sin x_\tau \tag{5}$$

$$\dot{y}=-\alpha y-\beta\sin x, \tag{6}$$

*Present address: Fakultät für Physik, Universität Freiburg, D-79104 Freiburg. Electronic address: hv@physik.uni-freiburg.de

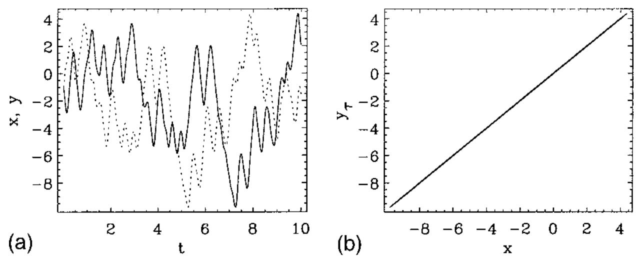

FIG. 1. Numerically simulated time series $x(t)$ (full line) and $y(t)$ (dotted line) (a) and the synchronization manifold between $x(t)$ and $y(t-\tau)$ (b). The response variable $y(t)$ anticipates the driver variable $x(t)$ with a time shift of $\tau=2$.

where $\alpha,\beta>0$. This model, describing phase shifts in nonlinear optics [10], is a well investigated example for delay-induced chaos [4]. With a Runge-Kutta integrator for delay-differential equations with a time step of $\Delta t=0.0025$, we simulate 4000 points each for x and y for the parameters $\tau=2$, $\alpha=1$, $\beta=20$, and random initial conditions [Fig. 1(a)]. For these parameters the drive system is well inside the chaotic regime [4]. The response variable y is shifted two time units to the left [Fig. 1(a)], thus anticipating the chaotic driver x. This can also be seen in the synchronization manifold [Fig. 1(b)] and the large value of the correlation coefficient R between trajectories $x(t)$ and $y(t-\tau)$: $R(x,y_\tau)>0.9999$. To show that this result is generic in the sense that it is robust with respect to small mismatches between the two systems, the parameters of the response system are disturbed, and the values for which $R(x,y_\tau)$ is still larger than 0.9 are searched for. These values are $\alpha_{\mathrm{resp}}\approx 0.4,\ldots,2.4$ and $\beta_{\mathrm{resp}}>0$ (the response y simply rescales like $y\to\beta_{\mathrm{resp}}/\beta\, y$). The absolute value of an additionally introduced phase shift of the sine function has to be smaller than 0.12π. For a phase outside this regime, one also observes antisynchronization, but no unstable solutions. Therefore, anticipating synchronization is extremely robust in this example.

These numerical results are in complete accordance with a simple perturbation analysis of the general system equations themselves: Consider, for arbitrary continuous f, a disturbed response system (3),

$$\dot{y}=-\alpha y+f(x)+\varepsilon g(x), \tag{7}$$

with a bounded continuous function $g(x)$. The general solution is

$$y(t)=e^{-\alpha t}y_0+\int_0^t e^{\alpha(u-t)}f(x(u))du + \varepsilon\int_0^t e^{\alpha(u-t)}g(x(u))du, \tag{8}$$

where $y_0=y(t=0)$. Now,

$$\Delta(t)=x(t)-y(t-\tau)=e^{-\alpha t}(x_0-e^{\alpha\tau}y_0) + \int_0^t e^{\alpha(u-t)}f(x(u-\tau))du-\int_0^t e^{\alpha(u-t)}f(x(u-\tau))du - \varepsilon\int_0^t e^{\alpha(u-t)}g(x(u-\tau))du. \tag{9}$$

For large t, one has, asymptotically,

$$\lim_{t\to\infty}|\Delta(t)|=\varepsilon\lim_{t\to\infty}\left|\int_0^t e^{\alpha(u-t)}g(x(u-\tau))du\right| \leqslant \varepsilon\sup_t|g(x(t))|\lim_{t\to\infty}\left|\int_0^t e^{\alpha(u-t)}du\right| = \frac{\varepsilon}{\alpha}\sup_t|g(x(t))|. \tag{10}$$

A similar result can be derived for a disturbance in the damping term $-\alpha y$. Therefore, after some transient time, small differences in the systems lead only to small deviations in the trajectories; anticipating synchronization is robust.

III. TIME-DELAYED DISSIPATIVE COUPLING

As a second coupling possibility, for the time being system (1) is coupled to a response system via Eq. (2). In Ref. [5] local stability of the synchronization manifold has been proved for a system of the form $\dot{x}=F(x,x_\tau)$, if it is coupled to a response system in the following way:

$$\dot{x}=F(x,x_\tau,p_0),$$
$$\dot{y}=F(y,y_\tau,p_0+K(x-y)). \tag{11}$$

This system depends on a perturbation parameter p that equals p_0 for the driver and $p_0+K(x-y)$ for the response system. In the synchronized state $x=y$ both parameters coincide. Using $r=-(\partial_x-K\partial_p)F(x,x_\tau,p_0)$ and $s=\partial_{x_\tau}F(x,x_\tau,p_0)$, the time evolution of the difference system with state variable $\Delta=x-y$ can be written as

$$\dot{\Delta}=-r\Delta+s\Delta_\tau \tag{12}$$

for small Δ, and the synchronization manifold is locally attracting if the origin of this equation is stable. Defining a suitable Lyapunov functional for the difference system, a sufficient synchronization condition is $r(t)>|s(t)|$ [11]. No assumptions are made about the specific form of $r(t)$ and $s(t)$. Even if one does not know the explicit time dependencies of $r(t)$ and $s(t)$ (for this one would need the solution of the system), in certain cases it is possible to estimate bounds for $r(t)$ and $s(t)$ by an analysis of the model equations (11).

For example, applying this procedure to Eqs. (5) and

$$\dot{y}=-\alpha y-\beta\sin y_\tau+K(x-y), \tag{13}$$

one obtains the sufficient synchronization condition

$$K>\beta-\alpha. \tag{14}$$

Next, we couple the systems by their delayed state variables x_τ and y_τ, i.e., the response is

$$\dot{y}=-\alpha y-\beta\sin y_\tau+K(x_\tau-y_\tau). \tag{15}$$

Using $r=-\partial_x F(x,x_\tau,p_0)$ and $s=(\partial_{x_\tau}-K\partial_p)F(x,x_\tau,p_0)$, one ends up with the same difference system $\dot{\Delta}=-r\Delta$

$+s\Delta_\tau$, and, with $|s(t)|=|(\beta+K)\cos(x_\tau)|\leqslant|\beta+K|$, a sufficient synchronization condition becomes

$$-\beta-\alpha<K<-\beta+\alpha. \tag{16}$$

In contrast to nondelayed coupling [Eq. (14)], K is also limited from above and may attain negative values.

This condition has been checked again numerically, with similar results as above. However, it is also observed that the synchronization manifold is only locally attractive; two systems with overly different initial conditions may not be attracted by the synchronization manifold before the response becomes unstable. The reason is that linear system (12) constitutes only an approximation of the transverse system for small Δ. This instability has not been observed numerically in the nondelayed coupling case of Eq. (13). However, if the coupling in system (15) is modified so as to yield the response system

$$\dot{y}=-\alpha y-\beta\sin y_\tau+K(\sin x_\tau-\sin y_\tau) \tag{17}$$

numerically, instabilities no longer seem to occur for any value of K. For $K=-\beta$ one again has coupling through complete replacement which is globally stable in any case, and for $K\neq-\beta$ the same local synchronization condition [Eq. (16)] as in the former case holds. This can be seen by linearizing the corresponding generalization of Eq. (11), and proceeding as above.

Since conditions (14) and (16) are only sufficient but not necessary for synchronization, in practice the allowed regime for K may be larger than stated. This is also observed in simulation studies. Note that for the limit of a vanishing delay, where Eqs. (13) and (15) become identical, the simultaneous existence of the two synchronization conditions (14) and (16) poses no contradiction, since in this case both systems exhibit only trivial dynamics and are synchronized for arbitrary coupling constants.

The system composed of Eqs. (5) and (17) resembles the system of two coupled phase-locked loops with a time delay (see the Appendix). Phase-locked loops play an important role in consumer electronics and physiological modeling [12,13].

IV. DRIVE SYSTEMS WITHOUT A MEMORY

Anticipating synchronization can also occur in continuous chaotic systems without a time delayed feedback in the driver. In this case, the chaotic flow of each subsystem is described by a system of at least three ordinary differential equations, and stability criteria are much harder to achieve than in the former cases of one-dimensional equations. Therefore, we restrict ourselves to proving the existence of an invariant anticipating manifold. The remaining question is then the stability of this manifold, which will be analyzed numerically.

As an example, two dissipatively coupled Rössler [14] systems are investigated, that is

$$\begin{aligned}
\dot{x}_1&=-x_2-x_3,\\
\dot{x}_2&=x_1+ax_2,\\
\dot{x}_3&=b+x_3(x_1-c),\\
\dot{y}_1&=-y_2-y_3+K(x_1-y_{1,\tau}),\\
\dot{y}_2&=y_1+ay_2,\\
\dot{y}_3&=b+y_3(y_1-c).
\end{aligned} \tag{18}$$

A numerical stability analysis for $\tau=0$ was performed recently [6], with the result that these systems can exhibit complete synchronization for a wide range of coupling constants. By introducing the specific coupling term $K(x_1-y_{1,\tau})$, for $\tau\neq0$ complete synchronization becomes impossible, because the manifold $(x_1,x_2,x_3)=(y_1,y_2,y_3)$ (or $\mathbf{x}=\mathbf{y}$) loses its invariance property. However, now defining the transversal system on an anticipating manifold, that is, $\mathbf{\Delta}:=\mathbf{x}-\mathbf{y}_\tau$, it becomes

$$\begin{aligned}
\dot{\Delta}_1&=-\Delta_2-\Delta_3-K\Delta_{1,\tau},\\
\dot{\Delta}_2&=\Delta_1+a\Delta_2,\\
\dot{\Delta}_3&=(y_{1,\tau}-c)\Delta_3+x_3\Delta_1.
\end{aligned} \tag{19}$$

Obviously, for *any* time delay τ, $\mathbf{\Delta}=\mathbf{0}$ is a fixed point of the coupled systems.

To judge the existence of anticipating chaotic synchronization for this system of linear nonautonomous delay-differential equations, the question of stability of the anticipating manifold has to be considered. Note that it is not assumed in Eq. (19) that $\mathbf{\Delta}$ is small. Surely, for very small anticipation times τ, synchronization with anticipating coupling should be stable for coupling constants for which the usually coupled systems (with $\tau=0$) are stable, since otherwise complete synchronization would not be a robust property of coupled Rössler systems. On the other hand, it is also sure that in contrast to the considerations in the previous two sections, the anticipation time must be limited, since otherwise one could anticipate a chaotic system for arbitrary spaces of time. The question is the following: How far can the time delay be increased, depending on the coupling constant, such that anticipating synchronization remains at least locally stable? Unfortunately, even for a vanishing anticipation time the stability regime in terms of the coupling constant can only be estimated numerically [6], and we are not aware of any rigorous results. More important, stability criteria for linear nonautonomous delay-differential equations are most often based on Lyapunov functionals, yielding delay-independent stability results. These methods [11] are therefore of little use here. The delay-dependent criteria of Ref. [15] are also far too conservative to give useful results for the present case.

To analyze the stability of the synchronization manifold of the coupled Rössler systems [Eqs. (18)], we compute trajectories for different coupling strengths K and time delays τ in the anticipating coupling term $K(x_1-y_{1,\tau})$. The correlation coefficients between these trajectories are shown in Fig. 2. As one can see, anticipating synchronization seems to be stable for anticipation times up to $\tau=0.8$. That is, for two coupled Rössler oscillators the dynamics of the chaotic

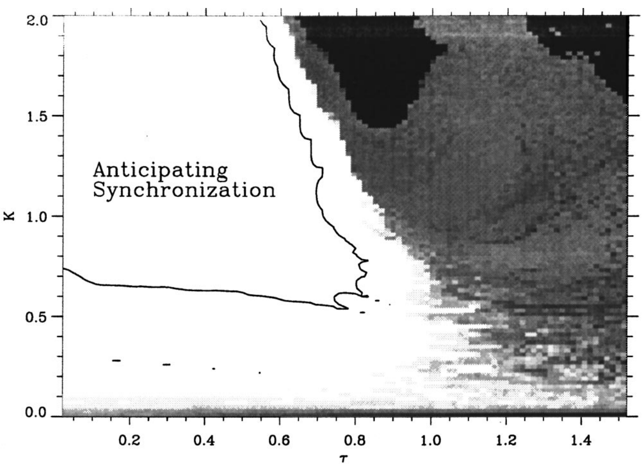

FIG. 2. A numerically estimated stability diagram for the coupled Rössler systems [Eq. (18)]. Light (dark) gray values correspond to large (small) correlation coefficients between the drive and response trajectories. The black regimes denote where the response system becomes unstable, and its trajectory is escaping to infinity. Additionally, contour lines for a correlation coefficient of $R = 0.9999$ are shown. In the area labeled ''anticipating synchronization,'' the correlation coefficient exceeds 0.9999. For the corresponding parameter values the synchronization manifolds resemble the one of Fig. 1(b). The model coefficients are $a=0.15$, $b=0.2$, and $c=10$, the integration time step is 0.05, and the data samples contain 4000 points with random initial conditions.

driver can be anticipated for more than one eighth of its intrinsic chaotic period of about $T=6.0$.

It would be interesting to relate the maximum anticipation time of a chaotic system without a time delay to other properties, like its spectrum of Lyapunov exponents and dissipation rate. Surely, the results would also depend strongly on the kind of coupling, which makes these questions rather formidable, and are beyond the scope of this paper. Here we have used only scalar dissipative coupling. However, it can be expected that the maximum anticipation time can be enlarged considerably by using nonscalar couplings or couplings with saturation terms to suppress the instability of the response. In the above considered case of two coupled phase-locked loops, the latter case already turned out to be effective, as discussed in the context of Eqs. (15) and (17).

V. DISCUSSION

We have given analytic evidence that dissipative chaotic systems with time-delayed feedback can force identical systems onto a synchronization manifold that involves the future state of the drive system, and that those systems can synchronize in the usual sense even if the coupling between both lies in the past. These results are counterintuitive, since in both cases the future evolution of the drive system is anticipated. This is most evident in the first case, ''anticipating synchronization,'' where the response system anticipates the drive system, just by synchronizing to a state that lies in the *future* of the driving system.

System (1) is the simplest case, containing both dissipation and memory. Since the condition for anticipating synchronization [Eq. (4)] does not depend on the specific nonlinearity *f*, the described phenomena are caused only by the interplay of dissipation and memory, and are thus of universal nature. (This phenomenon is completely different from, and has other explanations than, ''lag synchronization'' [16], where bidirectional coupling is considered.) The scope of the memory of the drive system, τ, determines how far the response system can anticipate the dynamics, but the synchronization conditions are independent of τ.

Anticipating synchronization can also occur in continuous chaotic systems without a memory in the drive system. This proves that the phenomenon is much more nontrivial, as might be concluded from the coupling of systems where the instability leading to chaos is due to a time delay. Dissipation and delayed feedback are rather common in nature, and we expect our results to be of some broader applicability:

For anticipating synchronization, the drive system can produce arbitrarily high-dimensional (thus complex) dynamics; it has been shown that in the asymptotic case of an infinitely large time delay or parameter β in Eq. (5), its solution can be completely described by means of a stochastic process [3]. Anticipating synchronization can be applied for a *fast prediction*, i.e., without any computation involved, of system (1) by simply coupling a near-identical system to it. This could be of advantage in fast electronic or optical devices, as they appear in communication systems [17] (also see the Appendix).

An immediate consequence of synchronization with delayed coupling is that systems which are separated by some distance can still synchronize, even if the signal transmission is slow and the coupling only one way. Since the underlying mechanisms are so simple, it should be worth searching for synchronization in physiological systems, where delayed feedback dynamics seem to play a crucial role [18]. In particular, arrays of phase-locked oscillators are suspected to be important for an understanding of neuronal information processing, and the introduction of a physiologically motivated time delay may improve such models [19].

It remains an open question if these phenomena can be exploited for *controlling* chaotic systems. This would be appealing since it would not be necessary to adjust the delay to the period of an unstable periodic orbit as in the method of time-delayed feedback control [20].

ACKNOWLEDGMENTS

We acknowledge the hospitality of the Georgia Institute of Technology and the University of Surrey, especially some hints from P. Ashwin, and financial support from the Max-Planck-Gesellschaft.

APPENDIX: COUPLED PHASE-LOCKED LOOPS

We derive the equation of motion for two coupled phase-locked loops (PLL's) with a time delay in the feedback loop. It will turn out that the coupling term has to be considered delayed in time, as in Eq. (17). The main components of a PLL with a time delay [13] are a reference oscillator producing a harmonic reference signal u_1; a controllable oscillator whose delayed output $u_{2,\tau}$ has to be synchronized with u_1; and a phase detector that mixes u_1 and $u_{2,\tau}$, to yield, after low-pass filtering, the signal u_3 that is fed back into the

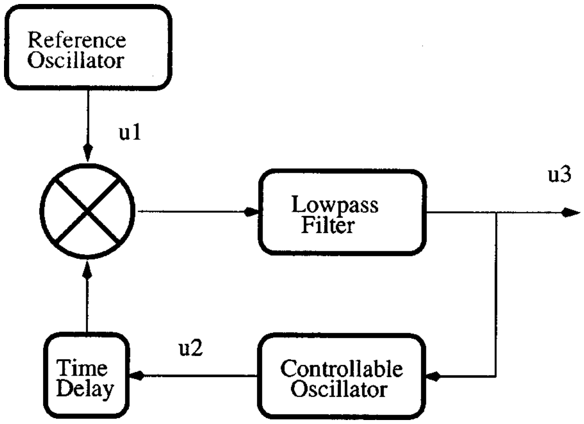

FIG. 3. A phase-locked loop with a time delay in the feedback loop.

controllable oscillator (Fig. 3).

To derive the equation of motion for a single PLL, assume that the reference oscillator produces a harmonic signal $u_1 = A\sin\Theta(t)$, with $\dot{\Theta}(t)=\omega_0$. The controllable oscillator produces the signal $u_2 = B\cos\Psi$, where the phase Ψ is controlled by the characteristic $\dot{\Psi}=\omega+\gamma u_3$. The signal u_1 and the delayed signal u_2, i.e., $u_{2,\tau}$, are now multiplied and rescaled. This gives the signal $\delta u_1 u_{2,\tau} = AB\delta/2(\sin(\Theta-\Psi_\tau)+\sin(\Theta+\Psi_\tau))$. The second sine function which is of higher frequency than the first one is subtracted by the low-pass filter, and one has $u_3 = AB\delta/2\sin(\Theta-\Psi_\tau)$, which is fed back into the controllable oscillator. Now the phase difference x between the reference oscillator signal u_1 and the delayed controllable oscillator signal $u_{2,\tau}$ evolves in time like $\dot{x}=\dot{\Theta}-\dot{\Psi}_\tau=\omega_0-\omega-AB\gamma\delta/2\sin(\Theta_\tau-\Psi_{2\tau})=-AB\gamma\delta/2\sin x_\tau+\omega_0-\omega$. With $\omega_0=\omega$ and $\beta=AB\gamma\delta/2$, this is Eq. (5) for $\alpha=0$.

Next, the equation of motion for two coupled PLL's is derived. To couple two PLL's with state variables u_i and v_i $(i=1,\ldots,3)$ in a physically reasonable way, we disturb the identical response system by the output u_3, such that the mean energy in the coupled system is conserved. Therefore, the signal v_3 is substituted by $v_3+K(u_3-v_3)$. Now for the response system the phase difference y between the common reference oscillator signal u_1 and the delayed controllable oscillator signal $v_{2,\tau}$ changes with time like $\dot{y}=\omega_0-\omega-AB\gamma\delta/2\sin y_\tau+K(\sin x_\tau-\sin y_\tau)$. With $\omega_0=\omega$ and $\beta=AB\gamma\delta/2$, this is Eq. (17) for $\alpha=0$.

[1] M. C. Mackey and L. Glass, Science **197**, 287 (1977).
[2] J. D. Farmer, Physica D **4**, 366 (1982).
[3] B. Dorizzi *et al.*, Phys. Rev. A **35**, 328 (1987).
[4] K. Ikeda and K. Matsumoto, Physica D **29**, 223 (1987).
[5] K. Pyragas, Phys. Rev. E **58**, 3067 (1998).
[6] L. M. Pecora *et al.*, Chaos **7**, 520 (1997).
[7] V. S. Afraimovich, N. N. Verichev, and M. I. Rabinovich, Radiophys. Quantum Electron. **29**, 795 (1986).
[8] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[9] S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (Springer, New York, 1990).
[10] K. Ikeda, H. Daido, and O. Akimoto, Phys. Rev. Lett. **45**, 709 (1980).
[11] I. Györy and G. Ladas, *Oscillation Theory of Delay Differential Equations* (Clarendon Press, Oxford, 1991); K. Gopalsami, *Stability and Oscillations in Delay Differential Equations of Population Dynamics* (Kluwer, Dordrecht, 1992); J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations* (Springer, New York, 1993); O. Diekmann *et al.*, *Delay Equations* (Springer, New York, 1995).
[12] *Phase-Locked Loops*, edited by W. C. Lindsey and C. M. Chie (IEEE Press, New York, 1986).
[13] W. Wischert, A. Wunderlin, and A. Pelster, Phys. Rev. E **49**, 203 (1994).
[14] O. E. Rössler, Phys. Lett. **47A**, 397 (1976).
[15] R. Volz, J. Math. Anal. Appl. **120**, 584 (1986); L. L. Ming, Math. Comput. Modelling **13**, 67 (1990).
[16] M. Rosenblum, A. S. Pikovsky, and J. Kurths, Phys. Rev. Lett. **78**, 4193 (1997).
[17] Delayed-feedback models can be realized in an optical manner [10] or electronically [A. Namajunas, K. Pyragas, and A. Tamaševičius, Int. J. Bifurcation Chaos Appl. Sci. Eng. **7**, 957 (1997)]. The notion of ''chaotic communication'' was described in K. M. Cuomo, A. V. Oppenheim, and S. H. Strogatz, IEEE Trans. Circuits Syst., II: Analog Digital Signal Process. **40**, 626 (1993).
[18] P. Tass *et al.*, Phys. Rev. E **54**, R2224 (1996); C. W. Eurich and J. G. Milton, *ibid.* **54**, 6681 (1996); *Nonlinear Analysis of Physiological Data*, edited by H. Kantz, J. Kurths, and G. Mayer-Kress (Springer, Berlin, 1998).
[19] P.A. Tass, *Phase Resetting in Medicine & Biology* (Springer, Berlin, 1999).
[20] W. Just *et al.*, Phys. Rev. Lett. **78**, 203 (1997); **81**, 562 (1998).